A Repair Framework for Scalar MDS Codes

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Main motivation

• Data in big companies like facebook are stored and managed in storage clusters involving lots of servers.

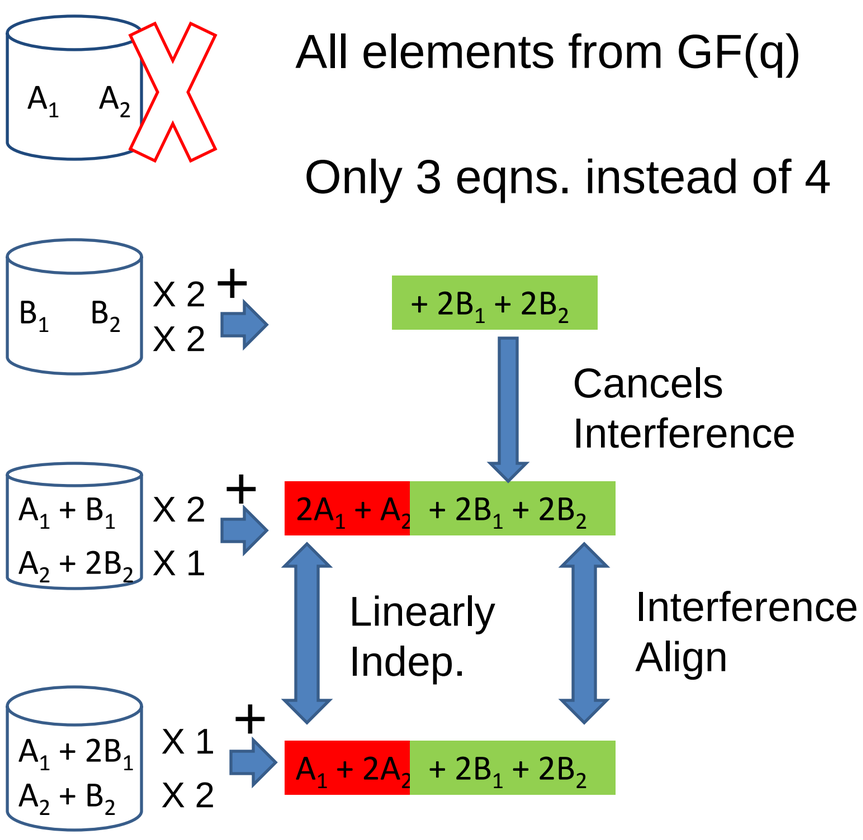
• To protect against failures: Reed Solomon like MDS codes used creating an (n,k) system.

• Most frequent type of failure: one node of out n nodes fail. Bandwidth for repair is constrained.

• Current theory focuses on vector codes to minimize bandwidth during one node failure.

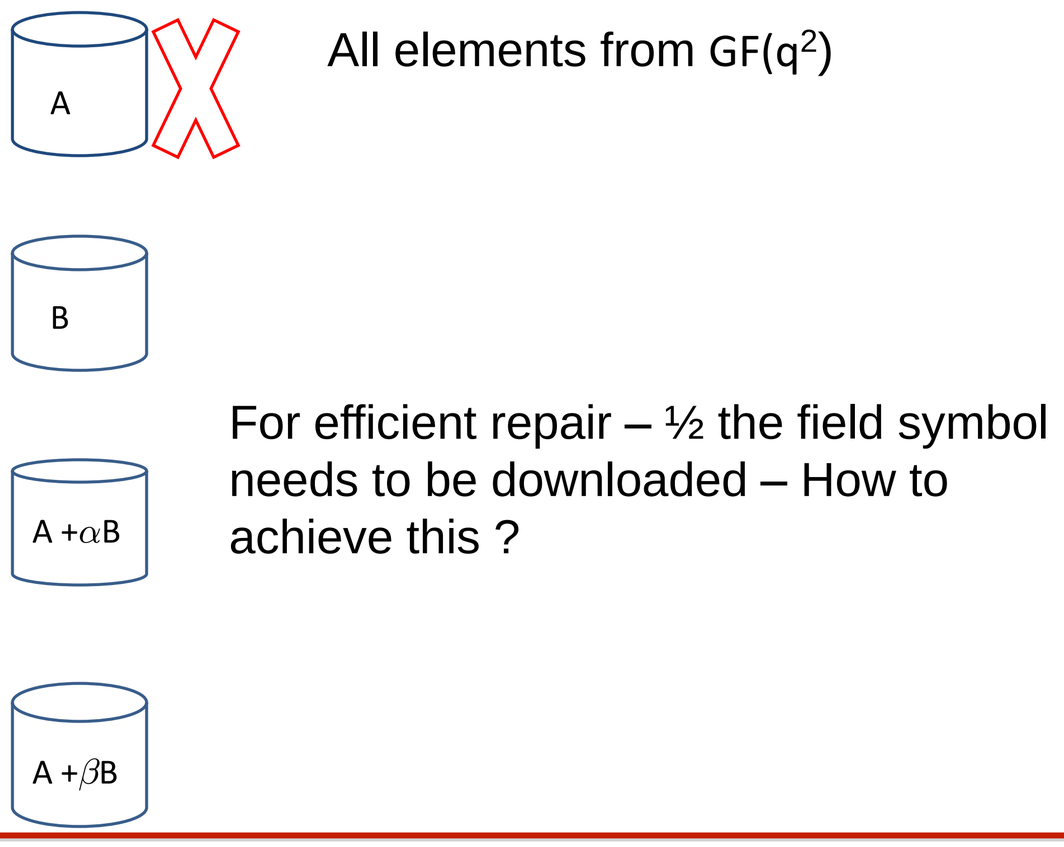
• We study how to repair scalar codes currently in deployment (Facebook employs (14,10) RS code !).

Vector Coded (n,k) system



1/2 unit of stored information is downloaded saving bandwidth

Scalar MDS (n,k) system



For efficient repair – ½ the field symbol needs to be downloaded – How to achieve this ?

Key Idea: Multiplication Operators

• For simplicity, let the code be over complex field $\mathbb{C}$ - an extension field of reals - $\mathbb{R}^2$.

$$A = A_1 + iA_2 \quad B = B_1 + iB_2 \quad \alpha = \alpha_1 + i\alpha_2$$

$$\alpha * B \equiv \quad \bar{\alpha} = \begin{bmatrix} \alpha_1 & -\alpha_2 \\ \alpha_2 & \alpha_1 \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$A + \alpha B \equiv \begin{bmatrix} 1 & 0 & \alpha_1 & -\alpha_2 \\ 0 & 1 & \alpha_2 & \alpha_1 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \end{bmatrix} \Rightarrow \text{Two Eqns.}$$

• Sub packetizing over GF(q), a scalar equation in GF(q^m) will have m equations – but more involved matrices.

Repair Field elements

$$[c_1 c_2] \times \begin{bmatrix} I \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} + \begin{bmatrix} \bar{\alpha} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$\underbrace{[d_1 d_2] \times \begin{bmatrix} I \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}}_{\text{Linearly Independent}} + \underbrace{\begin{bmatrix} \bar{\beta} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}}_{\text{Linearly Dependent}}$$

• There always exists a multiplication operator $\bar{\gamma}$ such that :

$$[c_1 c_2]\, \bar{\gamma} = \; [d_1 d_2]$$

Algebraic alignment condition

• 1 and γ are linearly indep. over the reals

• α and β are linearly dep. over the reals.

• Using this , one can come up with optimal repair scheme for any (n,n-2) RS code.

• Have efficient repair schemes for (5,3),(6,4) RS codes

References

1. K.Shanmugam, D.Papailiopoulos and A.Dimakis, “A Repair Framework for scalar MDS codes”, - to be submitted.
2. Y. Wu and A. G. Dimakis, “Reducing repair traffic for erasure coding-based storage via interference alignment,” in Proc. IEEE Int. Symp. on Information Theory (ISIT), Seoul, Korea, Jul. 2009.

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