Quantum measurement procedures via weak interactions

Jan Florjanczyk, Todd Brun

Communication Sciences Institute, Ming-Hsieh Department of Electrical Engineering

USC Viterbi School of Engineering

Overview

A weak quantum measurement is a measurement for which any outcome does not disturb the quantum state more than a small amount ε.

Weak measurements are universal: [OB05] showed one can construct a sequence of weak measurements that converge to any strong measurement using a random walk of weak measurement operators.

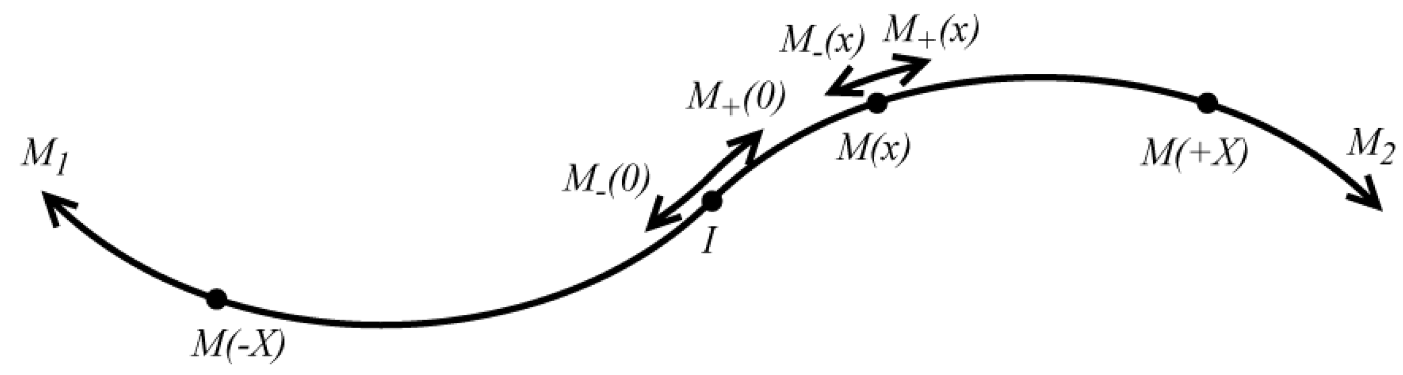
Weak measurement walks via an interacting probe: We study possible realizations of a such a procedure when the weak measurement is effectuated via weak interaction of a probe with the system in question.

Virtual measurements

Given a desired measurement $\{M_1, M_2\}$, we construct a parametrized weak measurement $\{M_\pm(x)\}$ such that we achieve the desired measurement in the continuous limit

$$\lim_{\delta\to 0} \prod_{j}^{\lfloor X/\delta \rfloor} M_+(j\delta) \propto M_1$$

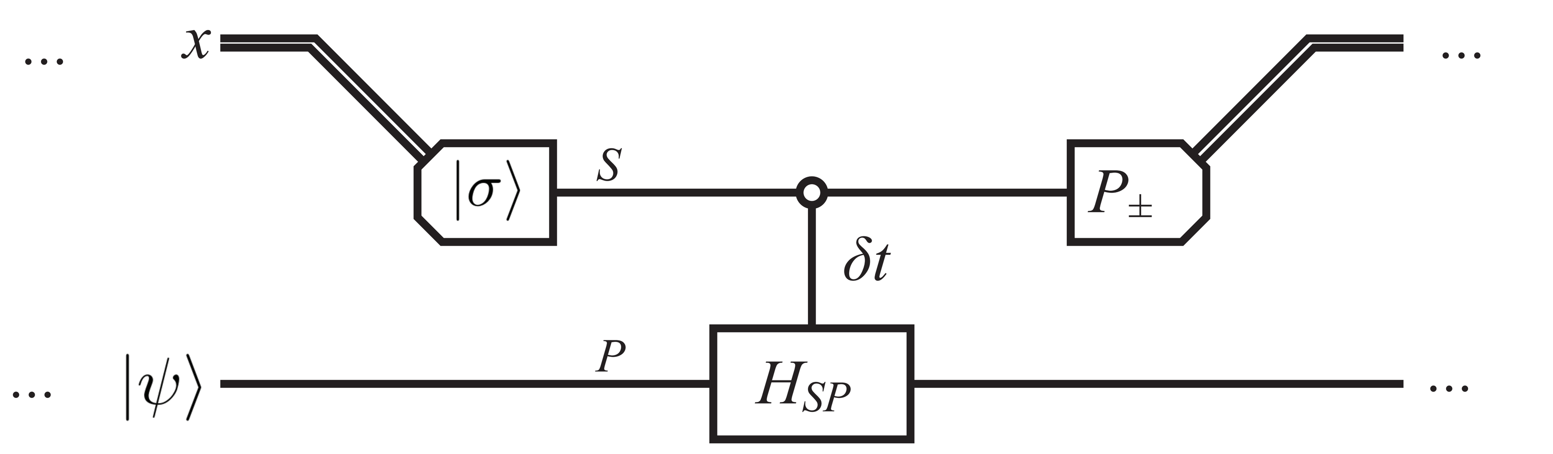
and similarly for M_- and M_2. This family of *step operators* $M_\pm(x)$ lives on a one-dimensional curve visualized below:



We call this the picture of *virtual measurements* since they do not involve our physical setup. Note that in order for the step operators to remain on the one-dimensional curve, we must also require that the $M_\pm(x)$ are proportional to the inverses of $M_\mp(x \pm \delta)$.

Physical measurements

The physical implementation of the step operators is achieved by preparing a probe state $|\sigma\rangle_P$, letting it interact with the target system $|\psi\rangle_S$, and measuring the probe via projective measurement $P_\pm$. One step of this procedure is visualized conceptually below



Note that the probe state is prepared depending on the position of the random walk x and the result of the projective measurement $P_\pm$ informs the next step of the walk. Altogether, if $P_\pm = |\pm\rangle\langle\pm|$, we have the following expression for the step operators

$$M_\pm(x) = \langle \pm | e^{-itH_{SP}} | \sigma(x) \rangle$$

where we've made the dependence of $|\sigma\rangle$ on x explicit. For short times t, the operator $M_\pm(x) \approx I$, the identity.

References

O. Oreshkov and T.A. Brun.
Weak measurements are universal.
Physical review letters, 95(11):110409, 2005.

The general diagonal measurement

We now present the control scheme that achieves any $M_1 = \mathrm{diag}\left(\sqrt{\alpha}, \sqrt{\beta}\right)$ and $M_2 = \mathrm{diag}\left(\sqrt{1-\alpha}, \sqrt{1-\beta}\right)$. In this control scheme we are given the interaction Hamiltonian $H_{SP} = Z_S \otimes Z_P$. Thus, both our probe and system are qubits. The procedure is as follows:

Initialize $x = 0$ and set

$$a_0 = \tanh^{-1}\left(\frac{\tanh\left(\frac{1}{2}\ln\frac{\alpha}{1-\alpha}\right)}{\tanh X}\right)$$

and similarly for b_0 and β.

Prepare a probe $|\sigma(x)\rangle$ in the state

$$|\sigma(x)\rangle = \sqrt{1/2 + p}|0\rangle + e^{-i\theta}\sqrt{1/2 - p}|1\rangle$$

where $p = 1/2\sqrt{1 - (\tanh(x - a_0) - \tanh(x - b_0))^2/4}$ and $\theta = -\mathrm{sgn}\left(\tanh(x - a_0) - \tanh(x - b_0)\right)\pi/2$.

Evolve Let the probe and system interact under H_{SP} for time $t = \delta$.

Measure the probe state using $P_\pm = |\phi^\pm\rangle\langle\phi^\pm|$ where

$$|\phi^\pm\rangle = \sqrt{1/2 \pm q}|0\rangle \pm \sqrt{1/2 \mp q}|1\rangle$$

and

$$q = \frac{\delta\left(\tanh(x - a_0) + \tanh(x - b_0)\right)}{2\sqrt{4 - (\tanh(x - a_0) - \tanh(x - b_0))^2}}$$

Update $x \leftarrow x \pm \delta$. Return to the **prepare** step.

Fixing a phase kick

Unfortunately, this procedure alone is insufficient to generate the M_1, M_2. In simulations and in calculations we find that our target state $|\psi\rangle$ rapidly accumulates a Z phase. We can choose to ignore this effect, undo it in one shot at the end of the procedure, or add the following steps after the **update** step above, before returning to the **prepare** step:

Prepare Given outcome $\pm$ from the projective measurement, prepare a probe state $|\sigma(x)\rangle$ as above with $p = \mp 1/2$

Evolve Let the probe and system interact under H_{SP} for time

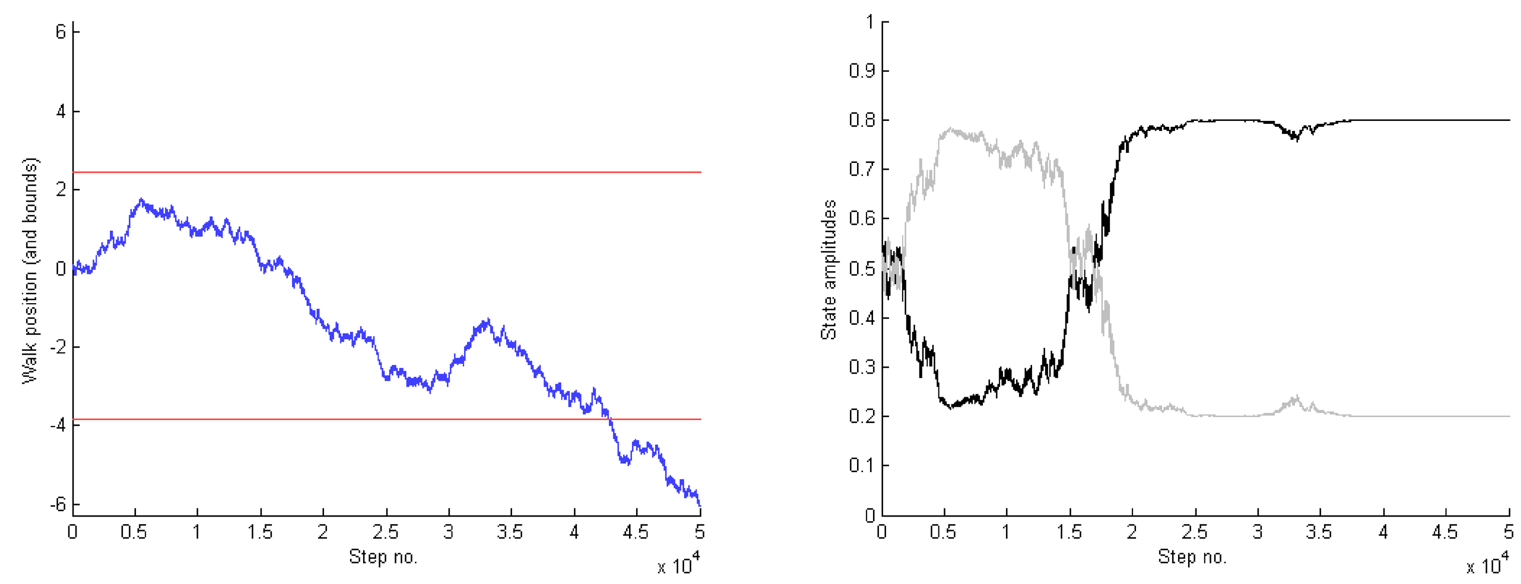
$$t = \frac{\delta(p + q)}{1/2 + 2pq + \sqrt{(1 - 4p^2)(1 - 4q^2)}\cos(\theta)}$$

where p, θ, and q are taken to be the values from above.

Return to the **prepare** step.

Example

We show an instance of a random walk of weak measurements for $\alpha = 0.8$, $\beta = 0.2$. **Left**: the walk position x and the bounds $(X - a_0)$, and $-(X - b_0)$. **Right**: the amplitudes of the state during the measurement procedure.



We run the walk for 5000 time-steps but in general, we can terminate the walk as soon as it exits one of the red boundaries. If we wish to ensure convergence with higher precision, we simply raise the value of X and spread the red boundary lines.