

Motivation

- Observing latent structure in a signal of interest allows for data analysis and interpretation.
- NMF is a widely-used method for observing latent structure, but is sensitive to noise.
- Noise in data prevents proper recovery of latent structure.
- **Goal:** Improve recovery of latent structure by jointly filtering the data to remove noise and factoring it to expose its latent structure.

Joint Filtering and Factorization

- Non-negative matrix factorization (NMF) provides a parts-based representation of a signal.
- Minimum variance distortionless response (MVDR) filter computes a power spectrum that estimates the spectral envelope of a signal without distorting the spectrum.
- Combine the cost functions of NMF and MVDR to perform joint filtering and factorization.
- $J = \underbrace{\|G \otimes V - WH\|_F^2}_{\text{Factorization}} + \lambda_1 \underbrace{\|G \otimes (WH)\|_F^2}_{\text{Filtering}} + \lambda_2 \underbrace{\|G \otimes (WH) - A \otimes (WH)\|_F^2}_{\text{Distortionless constraint}}$
- G filters the input signal V , which is factored into W and H .
- A is a frequency response that gives prior information about which frequency bands contain the signal of interest.
- Update equations:

$$G = \frac{(WH) \otimes V + \lambda_2 A \otimes (WH)^2}{V^2 + (\lambda_1 + \lambda_2)(WH)^2}$$

$$W \leftarrow W \otimes \frac{(G \otimes V)H^T}{WHH^T + \lambda_1 W \otimes C + \lambda_2 W \otimes D}$$

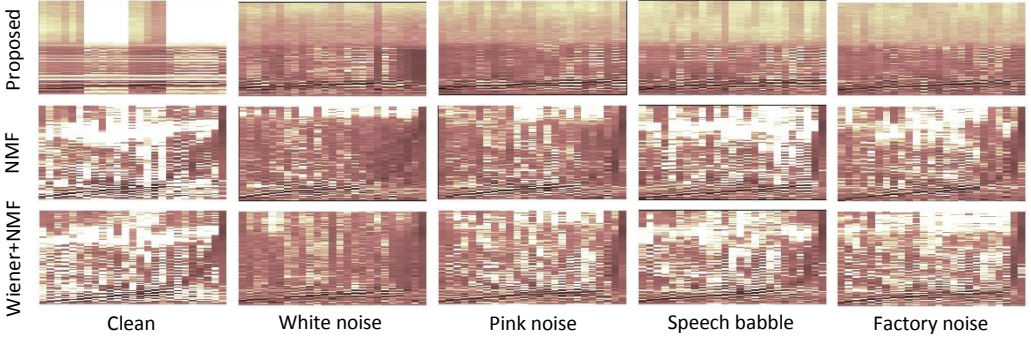
$$H \leftarrow H \otimes \frac{W^TWH + \lambda_1 E \otimes H + \lambda_2 F \otimes H}{W^T(G \otimes V)}$$
- Filtering operation can be viewed as computing the optimal filter $g_m[n]$ for each frame in the STFT of the noisy signal.

$$X(m, \omega) = \sum_{n=-\infty}^{\infty} x[n]w[n-m]e^{-j\omega n}$$

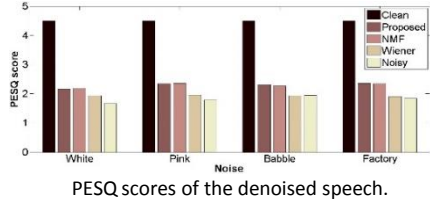
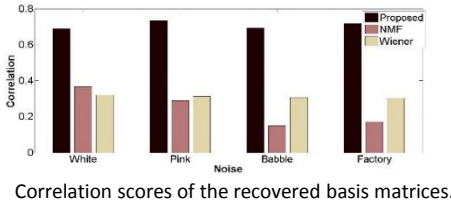
$$Y(m, \omega) = \sum_{n=-\infty}^{\infty} (x[n]w[n-m] * g_m[n])e^{-j\omega n}$$
 by Parseval's Theorem.

Basis Recovery Experiment

- Correlation metric: $\rho = \frac{1}{K} \sum_{k=1}^K \frac{|W_{\text{clean}}^T(:,k)W_{\text{noisy}}(:,k)|}{\|W_{\text{clean}}(:,k)\|_2 \|W_{\text{noisy}}(:,k)\|_2}$
- Added white, pink, speech babble, and factory noises to 100 TIMIT sentences at 5 dB and 10 dB SNR.



- Inverted WH (estimated clean spectrogram of the noisy speech) to obtain denoised speech.



Conclusion & Future Work

- Proposed joint filtering and factorization approach by combining NMF and MVDR into a single cost function.
- Correlation scores show better basis recovery of noisy signals.
- PESQ scores show denoising performance is comparable to other denoising methods.
- Use more generalized divergence metrics in cost function.
- Extend approach to other analysis methods, eg. PCA.
- Evaluate proposed algorithm's performance on other tasks, eg. phoneme recognition.